onnx_profile_ort

April 5, 2022

1 Profiling with onnxruntime

The notebook profiles the execution of an ONNX graph built from a *KMeans* model and executed with *onnxruntime*. It then study the decomposition of one einsum equation into more simple operators.

[1]:
```
from jyquickhelper import add_notebook_menu
add_notebook_menu()
```

[1]:
```
<IPython.core.display.HTML object>
```

[2]:
```
%matplotlib inline
```

[3]:
```
%load_ext mlprodict
```

1.1 KMeans

1.1.1 Builds a KMeans

[4]:
```
from sklearn.datasets import make_classification
X, y = make_classification(100000)
```

[5]:
```
from sklearn.cluster import KMeans
km = KMeans(max_iter=10)
km.fit(X)
```

[5]:
```
KMeans(max_iter=10)
```

[6]:
```
import numpy
from mlprodict.onnx_conv import to_onnx
onx = to_onnx(km, X[:1].astype(numpy.float32))
```

[7]:
```
%onnxview onx
```

[7]:
```
<jyquickhelper.jspy.render_nb_js_dot.RenderJsDot at 0x26206ad88b0>
```

1.1.2 Json

Another way to look into a model.

[8]:
```
from mlprodict.onnxrt import OnnxInference
```

```
oinf = OnnxInference(onx)
js = oinf.to_json()
```

[9]:
```
import json
from io import StringIO
from jyquickhelper import JSONJS
JSONJS(json.load(StringIO(oinf.to_json())))
```

[9]:
```
<jyquickhelper.jspy.render_nb_json.RenderJSON at 0x262341a3370>
```

1.1.3 Profiling

[10]:
```
from mlprodict.onnxrt import OnnxInference

oinf = OnnxInference(onx, runtime="onnxruntime1",
                     runtime_options={"enable_profiling": True})
```

[11]:
```
for i in range(0, 111):
    oinf.run({"X": X.astype(numpy.float32)})
```

[12]:
```
df = oinf.get_profiling(as_df=True)
df
```

[12]:

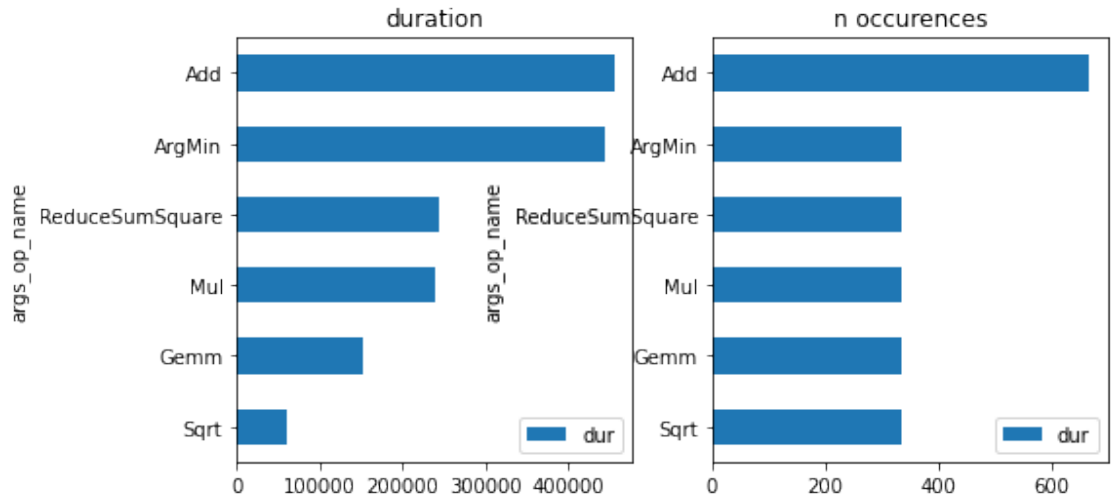
	cat	pid	tid	dur	ts	ph	\
0	Session	106368	299276	596	12	X	
1	Session	106368	299276	6925	670	X	
2	Node	106368	299276	1	34854	X	
3	Node	106368	299276	2939	34869	X	
4	Node	106368	299276	0	37872	X	
…	…	…	…	…	…	..	
2550	Node	106368	299276	0	2394227	X	
2551	Node	106368	299276	3511	2394228	X	
2552	Node	106368	299276	0	2397761	X	
2553	Session	106368	299276	11774	2385990	X	
2554	Session	106368	299276	11789	2385982	X	

	name	args_op_name	args_provider	\
0	model_loading_array	NaN	NaN	
1	session_initialization	NaN	NaN	
2	Re_ReduceSumSquare_fence_before	ReduceSumSquare	NaN	
3	Re_ReduceSumSquare_kernel_time	ReduceSumSquare	CPUExecutionProvider	
4	Re_ReduceSumSquare_fence_after	ReduceSumSquare	NaN	
…	…	…	…	
2550	Ar_ArgMin_fence_before	ArgMin	NaN	
2551	Ar_ArgMin_kernel_time	ArgMin	CPUExecutionProvider	
2552	Ar_ArgMin_fence_after	ArgMin	NaN	
2553	SequentialExecutor::Execute	NaN	NaN	
2554	model_run	NaN	NaN	

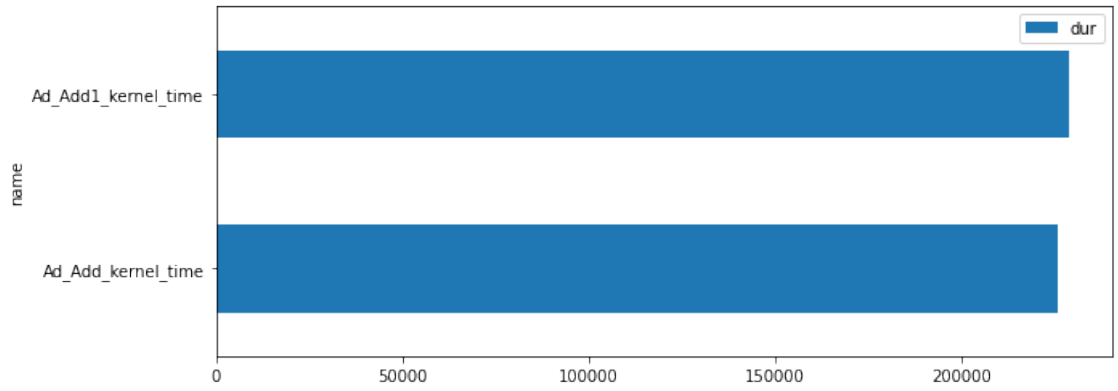
	args_graph_index	args_parameter_size	\
0	NaN	NaN	
1	NaN	NaN	
2	NaN	NaN	

```
3                    0                    0
4                  NaN                  NaN
…                  …                    …
2550               NaN                  NaN
2551                 5                    0
2552               NaN                  NaN
2553               NaN                  NaN
2554               NaN                  NaN

                           args_thread_scheduling_stats args_exec_plan_index  \
0                                                   NaN                   NaN
1                                                   NaN                   NaN
2                                                   NaN                   NaN
3     {'main_thread': {'thread_pool_name': 'session-…                     0
4                                                   NaN                   NaN
…                                                   …                     …
2550                                                NaN                   NaN
2551  {'main_thread': {'thread_pool_name': 'session-…                     5
2552                                                NaN                   NaN
2553                                                NaN                   NaN
2554                                                NaN                   NaN

     args_activation_size args_output_size
0                     NaN              NaN
1                     NaN              NaN
2                     NaN              NaN
3                 8000000           400000
4                     NaN              NaN
…                     …                …
2550                  NaN              NaN
2551              3200000           800000
2552                  NaN              NaN
2553                  NaN              NaN
2554                  NaN              NaN

[2555 rows x 15 columns]
```

[13]:
```
import matplotlib.pyplot as plt
gr_dur = df[['dur', "args_op_name"]].groupby("args_op_name").sum().sort_values('dur')
gr_n = df[['dur', "args_op_name"]].groupby("args_op_name").count().sort_values('dur')
gr_n = gr_n.loc[gr_dur.index, :]

fig, ax = plt.subplots(1, 2, figsize=(8, 4))
gr_dur.plot.barh(ax=ax[0])
gr_n.plot.barh(ax=ax[1])
ax[0].set_title("duration")
ax[1].set_title("n occurences");
```



[14]:
```
gr2 = df.loc[(df.args_op_name == 'Add') & (df.dur > 10), ['dur', "name"]].
  ↪groupby("name").sum().sort_values('dur')
gr2.plot.barh(figsize=(10, 4));
```



1.1.4 onnxruntime

[15]:
```
from onnxruntime import InferenceSession, RunOptions, SessionOptions
so = SessionOptions()
so.enable_profiling = True
sess = InferenceSession(onx.SerializeToString(), so)
```

[16]:
```
for i in range(0, 111):
    sess.run(None, {'X': X.astype(numpy.float32)}, )
```

[17]:
```
prof = sess.end_profiling()
prof
```

[17]:
```
'onnxruntime_profile__2021-05-13_13-58-59.json'
```

[18]:
```
with open(prof, "r") as f:
    js = json.load(f)

js[:3]
```

[18]:
```
[{'cat': 'Session',
  'pid': 106368,
  'tid': 299276,
  'dur': 450,
  'ts': 6,
  'ph': 'X',
  'name': 'model_loading_array',
  'args': {}},
 {'cat': 'Session',
  'pid': 106368,
  'tid': 299276,
  'dur': 3068,
  'ts': 498,
  'ph': 'X',
  'name': 'session_initialization',
  'args': {}},
 {'cat': 'Node',
  'pid': 106368,
  'tid': 299276,
  'dur': 1,
  'ts': 39069,
  'ph': 'X',
  'name': 'Re_ReduceSumSquare_fence_before',
  'args': {'op_name': 'ReduceSumSquare'}}]
```

[19]:
```
from pandas import DataFrame
from mlprodict.onnxrt.ops_whole.session import OnnxWholeSession

df = DataFrame(OnnxWholeSession.process_profiling(js))
df
```

[19]:

	cat	pid	tid	dur	ts	ph	\
0	Session	106368	299276	450	6	X	
1	Session	106368	299276	3068	498	X	
2	Node	106368	299276	1	39069	X	
3	Node	106368	299276	2804	39081	X	
4	Node	106368	299276	0	41947	X	
…	…	…	…	…	…	..	
2550	Node	106368	299276	0	2530548	X	
2551	Node	106368	299276	3501	2530550	X	
2552	Node	106368	299276	0	2534074	X	
2553	Session	106368	299276	14679	2519397	X	
2554	Session	106368	299276	14701	2519386	X	

	name	args_op_name	args_provider	\
0	model_loading_array	NaN	NaN	
1	session_initialization	NaN	NaN	
2	Re_ReduceSumSquare_fence_before	ReduceSumSquare	NaN	

```
3      Re_ReduceSumSquare_kernel_time  ReduceSumSquare  CPUExecutionProvider
4      Re_ReduceSumSquare_fence_after  ReduceSumSquare                   NaN
…                                  …               …                   …
2550           Ar_ArgMin_fence_before           ArgMin                   NaN
2551            Ar_ArgMin_kernel_time           ArgMin  CPUExecutionProvider
2552            Ar_ArgMin_fence_after           ArgMin                   NaN
2553      SequentialExecutor::Execute              NaN                   NaN
2554                        model_run              NaN                   NaN

     args_graph_index args_parameter_size  \
0                 NaN                 NaN
1                 NaN                 NaN
2                 NaN                 NaN
3                   0                   0
4                 NaN                 NaN
…                 …                   …
2550              NaN                 NaN
2551                5                   0
2552              NaN                 NaN
2553              NaN                 NaN
2554              NaN                 NaN

                          args_thread_scheduling_stats args_exec_plan_index  \
0                                                  NaN                   NaN
1                                                  NaN                   NaN
2                                                  NaN                   NaN
3     {'main_thread': {'thread_pool_name': 'session-…                     0
4                                                  NaN                   NaN
…                                                  …                     …
2550                                               NaN                   NaN
2551  {'main_thread': {'thread_pool_name': 'session-…                     5
2552                                               NaN                   NaN
2553                                               NaN                   NaN
2554                                               NaN                   NaN

     args_activation_size args_output_size
0                     NaN              NaN
1                     NaN              NaN
2                     NaN              NaN
3                 8000000           400000
4                     NaN              NaN
…                     …                …
2550                  NaN              NaN
2551              3200000           800000
2552                  NaN              NaN
2553                  NaN              NaN
2554                  NaN              NaN

[2555 rows x 15 columns]
```

1.2 Einsum: bsnh,btnh->bnts

This section looks into the ONNX graph produces by the decomposition of an einsum equation into more simple ONNX operator (no einsum).

1.2.1 Three implementations

[20]:
```
from mlprodict.testing.einsum import einsum as onx_einsum
from mlprodict.testing.einsum.einsum_fct import _einsum, enumerate_cached_einsum
from numpy import einsum as np_einsum
```

First classic numpy.

[21]:
```
equation = "bsnh,btnh->bnts"

N = 2
inputs = [numpy.random.randn(N, N, N, N).astype(numpy.float32),
          numpy.random.randn(N, N, N, N).astype(numpy.float32)]
np_einsum(equation, *inputs)
```

[21]:
```
array([[[[-2.373884  , -0.63942796],
         [ 1.0523143 ,  5.659873  ]],

        [[ 2.589915  , -0.18050319],
         [-0.6200199 ,  3.793615  ]]],


       [[[-0.37409338,  0.19822143],
         [ 1.2049038 ,  3.1882448 ]],

        [[-0.05218329,  0.87404007],
         [ 0.12789296,  1.4745121 ]]]], dtype=float32)
```

Then einsum executed by *onnxruntime*:

[22]:
```
onx_einsum(equation, *inputs, runtime='onnxruntime1', optimize=True, verbose=1,␣
 ↪decompose=False)
```

```
0.0026 best='sbhn,sthn->shtb': 100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 121/121 [00:01<00:00, 85.29it/
 ↪s]
```

[22]:
```
array([[[[-2.373884  , -0.63942796],
         [ 1.0523144 ,  5.659873  ]],

        [[ 2.589915  , -0.18050319],
         [-0.62002003,  3.793615  ]]],


       [[[-0.37409338,  0.19822143],
         [ 1.2049038 ,  3.1882448 ]],

        [[-0.05218329,  0.87404007],
         [ 0.12789296,  1.474512  ]]]], dtype=float32)
```

[23]:
```
obj = _einsum(equation, runtime='onnxruntime1', optimize=True, verbose=1,
              decompose=False, dtype=inputs[0].dtype)
```

[24]:
```
%onnxview obj.onnx_
```

[24]:
```
<jyquickhelper.jspy.render_nb_js_dot.RenderJsDot at 0x26237ce29a0>
```

Same equation but decomposed.

[25]:
```
obj = _einsum(equation, runtime='onnxruntime1', optimize=True, verbose=1,
              decompose=True, dtype=inputs[0].dtype)
```

```
0.0025 best='hsnt,hbnt->hnbs': 100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 121/121 [00:03<00:00, 34.54it/
↪s]
```

[26]:
```
%onnxview obj.onnx_
```

[26]:
```
<jyquickhelper.jspy.render_nb_js_dot.RenderJsDot at 0x2623b802df0>
```

[27]:
```
onx_einsum(equation, *inputs, runtime='onnxruntime1', optimize=True, verbose=1)
```

[27]:
```
array([[[[-2.373884  , -0.63942796],
         [ 1.0523144 ,  5.659873  ]],

        [[ 2.589915  , -0.18050319],
         [-0.62002003,  3.793615  ]]],


       [[[-0.37409338,  0.19822143],
         [ 1.2049038 ,  3.1882448 ]],

        [[-0.05218329,  0.87404007],
         [ 0.12789296,  1.474512  ]]]], dtype=float32)
```

1.2.2 First benchmark

[28]:
```
N = 20
inputs = [numpy.random.randn(N, N, N, N).astype(numpy.float32),
          numpy.random.randn(N, N, N, N).astype(numpy.float32)]
```

numpy.einsum

[29]:
```
%timeit numpy.einsum(equation, *inputs)
```

```
4.14 ms ± 350 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
```

onnxruntime einsum

[30]:
```
%timeit onx_einsum(equation, *inputs, runtime='onnxruntime1', optimize=True,␣
↪verbose=1, decompose=False)
```

```
736 µs ± 11.2 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

onnxruntime decomposed einsum

[31]:
```
%timeit onx_einsum(equation, *inputs, runtime='onnxruntime1', optimize=True, verbose=1)
```

```
525 µs ± 12.4 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

Let's disable the optimization to see the difference. The optimization goes through all the permutation of the letters of the equation and compares the computation time to find the best one.

[32]:
```
%timeit onx_einsum(equation, *inputs, runtime='onnxruntime1', optimize=False,␣
↪verbose=1, decompose=False)
```

```
761 µs ± 46.2 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

It has no significant impact here but it has for the decomposition. The not optimized version is much slower.

[33]:
```
%timeit onx_einsum(equation, *inputs, runtime='onnxruntime1', optimize=False,␣
↪verbose=1)
```

```
1.41 ms ± 43.1 µs per loop (mean ± std. dev. of 7 runs, 1000 loops each)
```

1.2.3 Profiling of the not optimized version

Let's profile the graph obtained with the decomposition.

[34]:
```
obj = _einsum(equation, runtime='onnxruntime1', optimize=False, verbose=1,
              decompose=True, dtype=inputs[0].dtype)
onx = obj.onnx_
```

[35]:
```
obj.equation, obj.equation_
```

[35]:
```
('bsnh,btnh->bnts', 'bsnh,btnh->bnts')
```

[36]:
```
from mlprodict.onnxrt import OnnxInference

oinf = OnnxInference(onx, runtime="onnxruntime1",
                     runtime_options={"enable_profiling": True})

d_inputs = {'X0': inputs[0], 'X1': inputs[1]}
for i in range(0, 100):
    oinf.run(d_inputs)

df = oinf.get_profiling(as_df=True)
df.head()
```

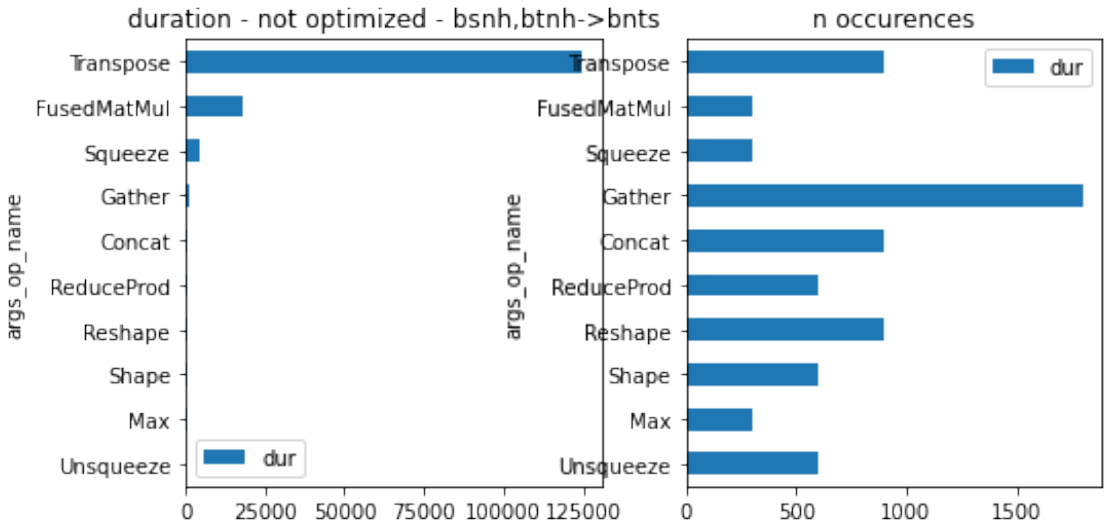
[36]:

	cat	pid	tid	dur	ts	ph	\
0	Session	106368	299276	705	4	X	
1	Session	106368	299276	7019	987	X	
2	Node	106368	299276	1	8320	X	
3	Node	106368	299276	4	8327	X	
4	Node	106368	299276	0	8372	X	

	name	args_op_name	args_provider	\
0	model_loading_array	NaN	NaN	
1	session_initialization	NaN	NaN	
2	Unsqueeze3_2620928306480_fence_before	Unsqueeze	NaN	

```
3   Unsqueeze3_2620928306480_kernel_time    Unsqueeze  CPUExecutionProvider
4   Unsqueeze3_2620928306480_fence_after    Unsqueeze                   NaN

  args_graph_index args_parameter_size  \
0              NaN                 NaN
1              NaN                 NaN
2              NaN                 NaN
3                4                   8
4              NaN                 NaN

                      args_thread_scheduling_stats args_exec_plan_index  \
0                                              NaN                  NaN
1                                              NaN                  NaN
2                                              NaN                  NaN
3  {'main_thread': {'thread_pool_name': 'session-…                    4
4                                              NaN                  NaN

  args_activation_size args_output_size
0                  NaN              NaN
1                  NaN              NaN
2                  NaN              NaN
3               640000           640000
4                  NaN              NaN
```

[37]:
```
import matplotlib.pyplot as plt
gr_dur = df[['dur', "args_op_name"]].groupby("args_op_name").sum().sort_values('dur')
gr_n = df[['dur', "args_op_name"]].groupby("args_op_name").count().sort_values('dur')
gr_n = gr_n.loc[gr_dur.index, :]

fig, ax = plt.subplots(1, 2, figsize=(8, 4))
gr_dur.plot.barh(ax=ax[0])
gr_n.plot.barh(ax=ax[1])
ax[0].set_title("duration - not optimized - %s" % obj.equation_)
ax[1].set_title("n occurences");
```



1.2.4 Profiling of the optimized version

[38]:
```
obj = _einsum(equation, runtime='onnxruntime1', optimize=True, verbose=1,
              decompose=True, dtype=inputs[0].dtype)
onx = obj.onnx_
```

[39]:
```
obj.equation, obj.equation_
```

[39]:
```
('bsnh,btnh->bnts', 'hsnt,hbnt->hnbs')
```

The second equation is the optimized equation.

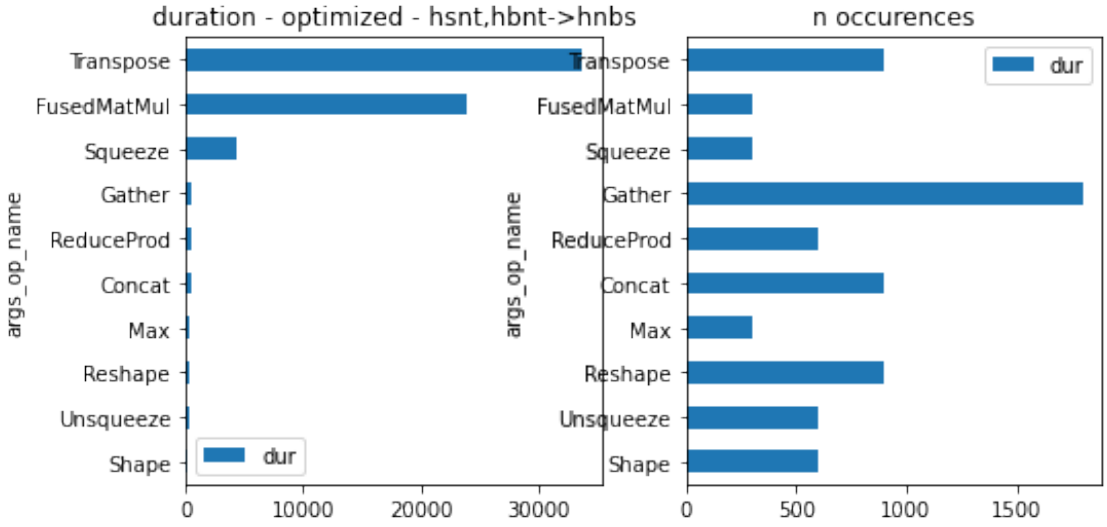
[40]:
```
from mlprodict.onnxrt import OnnxInference

oinf = OnnxInference(onx, runtime="onnxruntime1",
                     runtime_options={"enable_profiling": True})

d_inputs = {'X0': inputs[0], 'X1': inputs[1]}
for i in range(0, 100):
    oinf.run(d_inputs)

df = oinf.get_profiling(as_df=True)
df.head()
```

[40]:

	cat	pid	tid	dur	ts	ph	name	args_op_name	args_provider	args_graph_index	args_parameter_size	args_thread_scheduling_stats	args_exec_plan_index	args_activation_size	args_output_size
0	Session	106368	299276	1300	6	X	model_loading_array	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
1	Session	106368	299276	7330	1720	X	session_initialization	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2	Node	106368	299276	1	9376	X	Unsqueeze3_2620928202160_fence_before	Unsqueeze	NaN	NaN	NaN	NaN	NaN	NaN	NaN
3	Node	106368	299276	4	9383	X	Unsqueeze3_2620928202160_kernel_time	Unsqueeze	CPUExecutionProvider	4	8	{'main_thread': {'thread_pool_name': 'session-…	4	NaN	NaN
4	Node	106368	299276	0	9422	X	Unsqueeze3_2620928202160_fence_after	Unsqueeze	NaN	NaN	NaN	NaN	NaN	NaN	NaN

```
0                NaN              NaN
1                NaN              NaN
2                NaN              NaN
3             640000           640000
4                NaN              NaN
```

[41]:
```
gr_dur = df[['dur', "args_op_name"]].groupby("args_op_name").sum().sort_values('dur')
gr_n = df[['dur', "args_op_name"]].groupby("args_op_name").count().sort_values('dur')
gr_n = gr_n.loc[gr_dur.index, :]

fig, ax = plt.subplots(1, 2, figsize=(8, 4))
gr_dur.plot.barh(ax=ax[0])
gr_n.plot.barh(ax=ax[1])
ax[0].set_title("duration - optimized - %s" % obj.equation_)
ax[1].set_title("n occurences");
```



onnxruntime was able to fuse MatMul with a transposition. That explains why it is faster.

[42]:
```
gr_dur = df[['dur', "args_op_name", "name"]].groupby(["args_op_name", "name"],␣
  ↪as_index=False).sum().sort_values('dur')
gr_dur
```

[42]:
```
    args_op_name                                      name    dur
0         Concat                    Concat12_fence_after      0
24        Gather                     Gather1_fence_after      0
25        Gather                    Gather1_fence_before      0
27        Gather                      Gather_fence_after      0
60     Transpose  Transpose02134_2620928192768_fence_after      0
..           …                                        …    …
56       Squeeze       Squeeze4_2620928194352_kernel_time   4339
59     Transpose  Transpose01324_2620928151024_kernel_time   8661
62     Transpose  Transpose02134_2620928192768_kernel_time  11487
65     Transpose  Transpose13024_2620928192816_kernel_time  13598
11   FusedMatMul        MatMul_With_Transpose_kernel_time  23847
```

```
[72 rows x 3 columns]
```

[43]:
```
gr_dur[gr_dur.args_op_name == "Transpose"]
```

[43]:

	args_op_name	name	dur
60	Transpose	Transpose02134_2620928192768_fence_after	0
57	Transpose	Transpose01324_2620928151024_fence_after	0
61	Transpose	Transpose02134_2620928192768_fence_before	0
58	Transpose	Transpose01324_2620928151024_fence_before	1
64	Transpose	Transpose13024_2620928192816_fence_before	1
63	Transpose	Transpose13024_2620928192816_fence_after	3
59	Transpose	Transpose01324_2620928151024_kernel_time	8661
62	Transpose	Transpose02134_2620928192768_kernel_time	11487
65	Transpose	Transpose13024_2620928192816_kernel_time	13598

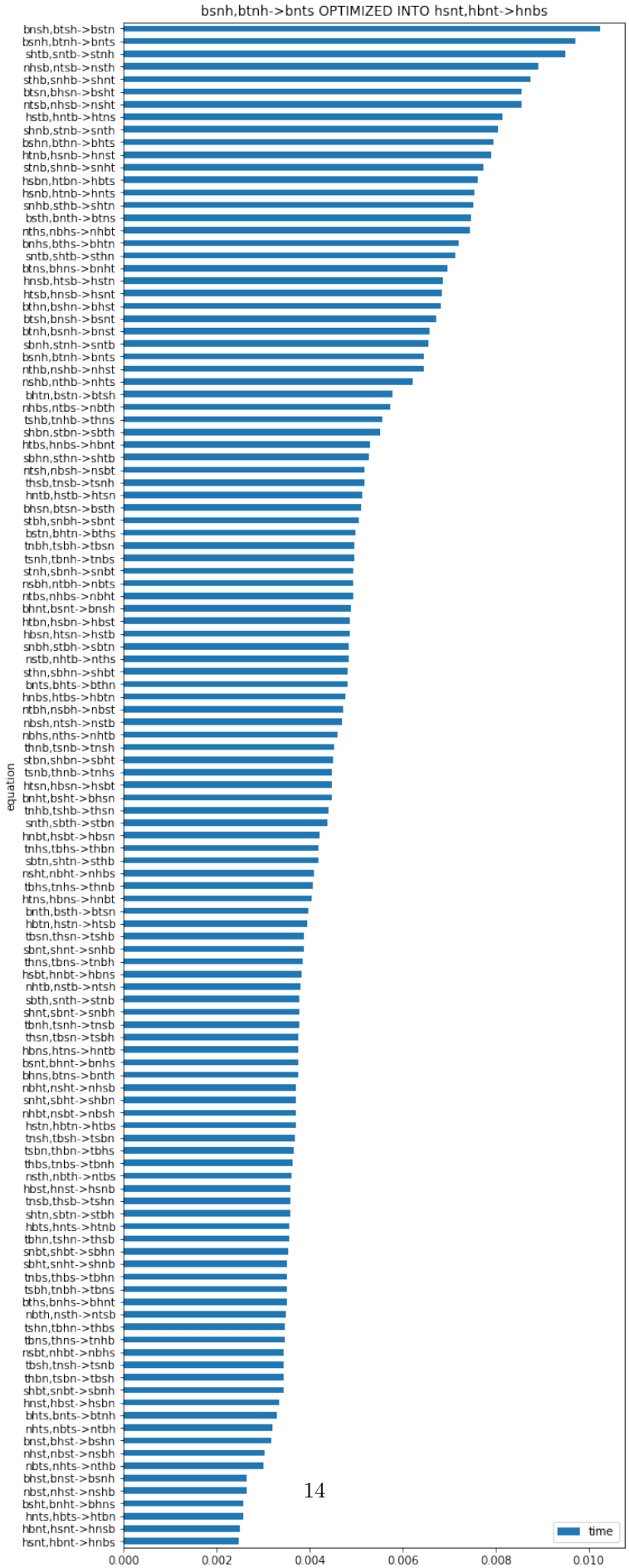
Let's draw again the graph to see which transpose is is which.

[44]:
```
%onnxview onx
```

[44]:
```
<jyquickhelper.jspy.render_nb_js_dot.RenderJsDot at 0x262366c7280>
```

The optimized looked into all permutations. We see that the letter ordering should be carefully chosen.

[45]:
```
import pandas
df = pandas.DataFrame(obj.timed_permutations_, columns=["time", "equation"])
df = df.sort_values('time')
df = df.set_index("equation")
ax = df.plot.barh(figsize=(8, 25))
ax.set_title("%s OPTIMIZED INTO %s" % (obj.equation, obj.equation_));
```


bsnh,btnh->bnts OPTIMIZED INTO hsnt,hbnt->hnbs
equation
bnsh,btsh->bstn
bsnh,btnh->bnts
shtb,sntb->stnh
nhsb,ntsb->nsth
sthb,snhb->shnt
btsn,bhsn->bsht
ntsb,nhsb->nsht
hstb,hntb->htns
shnb,stnb->snth
bshn,bthn->bhts
htnb,hsnb->hnst
stnb,shnb->snht
hsbn,htbn->hbts
hsnb,htnb->hnts
snhb,sthb->shtn
bsth,bnth->btns
nths,nbhs->nhbt
bnhs,bths->bhtn
sntb,shtb->sthn
btns,bhns->bnht
hnsb,htsb->hstn
htsb,hnsb->hsnt
bthn,bshn->bhst
btsh,bnsh->bsnt
btnh,bsnh->bnst
sbnh,stnh->sntb
bsnh,btnh->bnts
nthb,nshb->nhst
nshb,nthb->nhts
bhtn,bstn->btsh
nhbs,ntbs->nbth
tshb,tnhb->thns
shbn,stbn->sbth
htbs,hnbs->hbnt
sbhn,sthn->shtb
ntsh,nbsh->nsbt
thsb,tnsb->tsnh
hntb,hstb->htsn
bhsn,btsn->bsth
stbh,snbh->sbnt
bstn,bhtn->bths
tnbh,tsbh->tbsn
tsnh,tbnh->tnbs
stnh,sbnh->snbt
nsbh,ntbh->nbts
ntbs,nhbs->nbht
bhnt,bsnt->bnsh
htbn,hsbn->hbst
hbsn,htsn->hstb
snbh,stbh->sbtn
nstb,nhtb->nths
sthn,sbhn->shbt
bnts,bhts->bthn
hnbs,htbs->hbtn
ntbh,nsbh->nbst
nbsh,ntsh->nstb
nbhs,nths->nhtb
thnb,tsnb->tnsh
stbn,shbn->sbht
tsnb,thnb->tnhs
htsn,hbsn->hsbt
bnht,bsht->bhsn
tnhb,tshb->thsn
snth,sbth->stbn
hnbt,hsbt->hbsn
tnhs,tbhs->thbn
sbtn,shtn->sthb
nsht,nbht->nhbs
tbhs,tnhs->thnb
htns,hbns->hnbt
bnth,bsth->btsn
hbtn,hstn->htsb
tbsn,thsn->tshb
sbnt,shnt->snhb
thns,tbns->tnbh
hsbt,hnbt->hbns
nhtb,nstb->ntsh
sbth,snth->stnb
shnt,sbnt->snbh
tbnh,tsnh->tnsb
thsn,tbsn->tsbh
hbns,htns->hntb
bsnt,bhnt->bnhs
bhns,btns->bnth
nbht,nsht->nhsb
snht,sbht->shbn
nhbt,nsbt->nbsh
hstn,hbtn->htbs
tnsh,tbsh->tsbn
tsbn,thbn->tbhs
thbs,tnbs->tbnh
nsth,nbth->ntbs
hbst,hnst->hsnb
tnsb,thsb->tshn
shtn,sbtn->stbh
hbts,hnts->htnb
tbhn,tshn->thsb
snbt,shbt->sbhn
sbht,snht->shnb
tnbs,thbs->tbhn
tsbh,tnbh->tbns
bths,bnhs->bhnt
nbth,nsth->ntsb
tshn,tbhn->thbs
tbns,thns->tnhb
nsbt,nhbt->nbhs
tbsh,tnsh->tsnb
thbn,tsbn->tbsh
shbt,snbt->sbnh
hnst,hbst->hsbn
bhts,bnts->btnh
nhts,nbts->ntbh
bnst,bhst->bshn
nhst,nbst->nsbh
nbts,nhts->nthb
bhst,bnst->bsnh
nbst,nhst->nshb
bsht,bnht->bhns
hnts,hbts->htbn
hbnt,hsnt->hnsb
hsnt,hbnt->hnbs
time
0.000
0.002
0.004
0.006
0.008
0.010

[46]: