onnx_ffts

April 5, 2022

1 ONNX FFTs

Implementation of a couple of variations of FFT (see FFT in ONNX.

[1]:
```
from jyquickhelper import add_notebook_menu
add_notebook_menu()
```

[1]:
```
<IPython.core.display.HTML object>
```

[2]:
```
%matplotlib inline
```

[3]:
```
%load_ext mlprodict
```

1.1 Signature

We try to use function FFT or torch.fft.fftn.

[4]:
```
import numpy
from numpy.testing import assert_almost_equal

def numpy_fftn(x, fft_type, fft_length, axes):
    """
    Implements FFT

    :param x: input
    :param fft_type: string (see below)
    :param fft_length: length on each axis of axes
    :param axes: axes
    :return: result

    * `'FFT`': complex-to-complex FFT. Shape is unchanged.
    * `'IFFT`': Inverse complex-to-complex FFT. Shape is unchanged.
    * `'RFFT`': Forward real-to-complex FFT.
      Shape of the innermost axis is reduced to fft_length[-1] // 2 + 1 if␣
 ↪fft_length[-1]
      is a non-zero value, omitting the reversed conjugate part of
      the transformed signal beyond the Nyquist frequency.
    * `'IRFFT`': Inverse real-to-complex FFT (ie takes complex, returns real).
      Shape of the innermost axis is expanded to fft_length[-1] if fft_length[-1]
      is a non-zero value, inferring the part of the transformed signal beyond the␣
 ↪Nyquist
      frequency from the reverse conjugate of the 1 to fft_length[-1] // 2 + 1 entries.
```

```
    """
    if fft_type == 'FFT':
        return numpy.fft.fftn(x, fft_length, axes=axes)
    raise NotImplementedError("Not implemented for fft_type=%r." % fft_type)


def test_fct(fct1, fct2, fft_type='FFT', decimal=5):
    cases = list(range(4, 20))
    dims = [[c] for c in cases] + [[4,4,4,4], [4,5,6,7]]
    lengths_axes = [([c], [0]) for c in cases] + [
        ([2, 2, 2, 2], None), ([2, 6, 7, 2], None), ([2, 3, 4, 5], None),
        ([2], [3]), ([3], [2])]
    n_test = 0
    for ndim in range(1, 5):
        for dim in dims:
            for length, axes in lengths_axes:
                if axes is None:
                    axes = range(ndim)
                di = dim[:ndim]
                axes = [min(len(di) - 1, a) for a in axes]
                le = length[:ndim]
                if len(length) > len(di):
                    continue
                mat = numpy.random.randn(*di).astype(numpy.float32)
                try:
                    v1 = fct1(mat, fft_type, le, axes=axes)
                except Exception as e:
                    raise AssertionError(
                        "Unable to run %r mat.shape=%r ndim=%r di=%r fft_type=%r le=%r␣
 ↪"
                        "axes=%r exc=%r" %(
                            fct1, mat.shape, ndim, di, fft_type, le, axes, e))
                v2 = fct2(mat, fft_type, le, axes=axes)
                try:
                    assert_almost_equal(v1, v2, decimal=decimal)
                except AssertionError as e:
                    raise AssertionError(
                        "Failure mat.shape=%r, fft_type=%r, fft_length=%r" % (
                            mat.shape, fft_type, le)) from e
                n_test += 1
    return n_test


test_fct(numpy_fftn, numpy_fftn)
```

[4]: 1302

[5]:
```
%timeit -n 1 -r 1 test_fct(numpy_fftn, numpy_fftn)
```

```
1.81 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
```

[6]:
```
import torch
```

```
def torch_fftn(x, fft_type, fft_length, axes):
    xt = torch.tensor(x)
    if fft_type == 'FFT':
        return torch.fft.fftn(xt, fft_length, axes).cpu().detach().numpy()

%timeit -n 1 -r 1 test_fct(numpy_fftn, torch_fftn)
```

```
2.07 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
```

1.2 Numpy implementation

[7]:
```
import numpy


def _dft_cst(N, fft_length, dtype):
    def _arange(dim, dtype, resh):
        return numpy.arange(dim).astype(dtype).reshape(resh)

    def _prod(n, k):
        return (-2j * numpy.pi * k / fft_length) * n

    def _exp(m):
        return numpy.exp(m)

    n = _arange(N, dtype, (-1, 1))
    k = _arange(fft_length, dtype, (1, -1))
    M = _exp(_prod(n, k))
    return M


def custom_fft(x, fft_type, length, axis, dft_fct=None):
    # https://github.com/numpy/numpy/blob/4adc87dff15a247e417d50f10cc4def8e1c17a03/
 ↪numpy/fft/_pocketfft.py#L56
    if dft_fct is None:
        dft_fct = _dft_cst
    if fft_type == 'FFT':
        if x.shape[axis] > length:
            # fft_length > shape on the same axis
            # the matrix is shortened
            slices = [slice(None)] * len(x.shape)
            slices[axis] = slice(0, length)
            new_x = x[tuple(slices)]
        elif x.shape[axis] == length:
            new_x = x
        else:
            # other, the matrix is completed with zeros
            shape = list(x.shape)
            shape[axis] = length
            slices = [slice(None)] * len(x.shape)
            slices[axis] = slice(0, length)
            zeros = numpy.zeros(tuple(shape), dtype=x.dtype)
            index = [slice(0, i) for i in x.shape]
            zeros[tuple(index)] = x
```

```
            new_x = zeros

        cst = dft_fct(new_x.shape[axis], length, x.dtype)
        perm = numpy.arange(len(x.shape)).tolist()
        if perm[axis] == perm[-1]:
            res = numpy.matmul(new_x, cst).transpose(perm)
        else:
            perm[axis], perm[-1] = perm[-1], perm[axis]
            rest = new_x.transpose(perm)
            res = numpy.matmul(rest, cst).transpose(perm)
            perm[axis], perm[0] = perm[0], perm[axis]
        return res
    raise ValueError("Unexpected value for fft_type=%r." % fft_type)


def custom_fftn(x, fft_type, fft_length, axes, dft_fct=None):
    if len(axes) != len(fft_length):
        raise ValueError("Length mismatch axes=%r, fft_length=%r." % (
           axes, fft_length))
    if fft_type == 'FFT':
        res = x
        for i in range(len(fft_length) - 1, -1, -1):
            length = fft_length[i]
            axis = axes[i]
            res = custom_fft(res, fft_type, length, axis, dft_fct=dft_fct)
        return res
    raise ValueError("Unexpected value for fft_type=%r." % fft_type)


shape = (4, )
fft_length = [5,]
axes = [0]
rnd = numpy.random.randn(*shape) + numpy.random.randn(*shape) * 1j
custom_fftn(rnd, 'FFT', fft_length, axes), numpy_fftn(rnd, 'FFT', fft_length, axes)
assert_almost_equal(custom_fftn(rnd, 'FFT', fft_length, axes),
                    numpy_fftn(rnd, 'FFT', fft_length, axes), decimal=5)

shape = (4, 3)
fft_length = [3, 2]
axes = [0, 1]
rnd = numpy.random.randn(*shape) + numpy.random.randn(*shape) * 1j
custom_fftn(rnd, 'FFT', fft_length, axes), numpy_fftn(rnd, 'FFT', fft_length, axes)
assert_almost_equal(custom_fftn(rnd, 'FFT', fft_length, axes),
                    numpy_fftn(rnd, 'FFT', fft_length, axes), decimal=5)
```

[8]:
```
%timeit -n 1 -r 1 test_fct(numpy_fftn, custom_fftn, decimal=4)
```

```
2.35 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
```

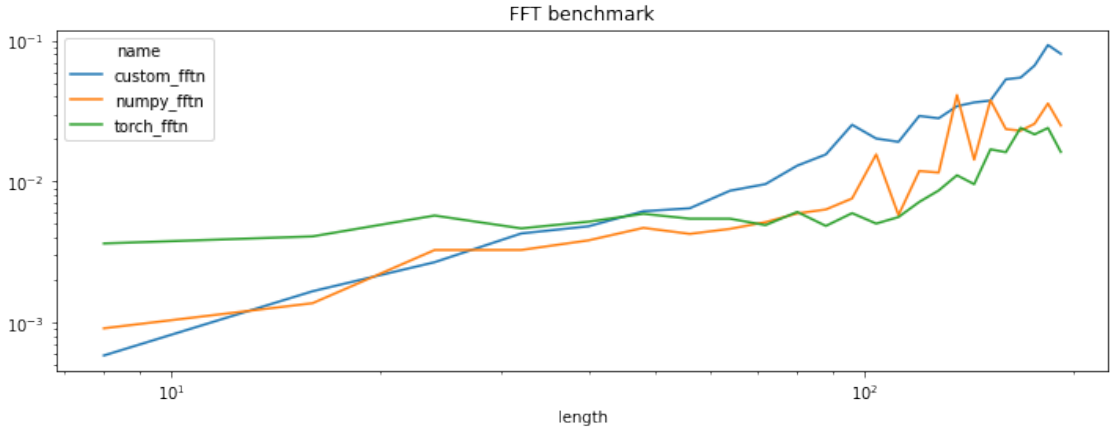
1.3 Benchmark

[9]:
```
from cpyquickhelper.numbers.speed_measure import measure_time
from tqdm import tqdm
from pandas import DataFrame

def benchmark(fcts, power2=False):
    axes = [1]
    if power2:
        shape = [512, 1024]
        lengths = [2, 4, 8, 16, 32, 64, 128, 256, 512, 1024]
    else:
        shape = [512, 150]
        lengths = list(range(8, 200, 8))
    rnd = numpy.random.randn(*shape) + numpy.random.randn(*shape) * 1j

    data = []
    for length in tqdm(lengths):
        fft_length = [length]
        for name, fct in fcts.items():
            obs = measure_time(lambda: fct(rnd, 'FFT', fft_length, axes),
                               repeat=5, number=5)
            obs['name'] = name
            obs['length'] = length
            data.append(obs)

    df = DataFrame(data)
    return df


df = benchmark({'numpy_fftn': numpy_fftn, 'custom_fftn': custom_fftn, 'torch_fftn':␣
 ↪torch_fftn})
piv = df.pivot("length", "name", "average")
piv[:5]
```

```
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 24/24 [00:06<00:00,  3.91it/s]
```

[9]:
```
name    custom_fftn  numpy_fftn  torch_fftn
length
8          0.000585    0.000911    0.003643
16         0.001669    0.001373    0.004087
24         0.002682    0.003273    0.005745
32         0.004288    0.003275    0.004657
40         0.004818    0.003831    0.005198
```

[10]:
```
piv.plot(logy=True, logx=True, title="FFT benchmark", figsize=(12, 4));
```



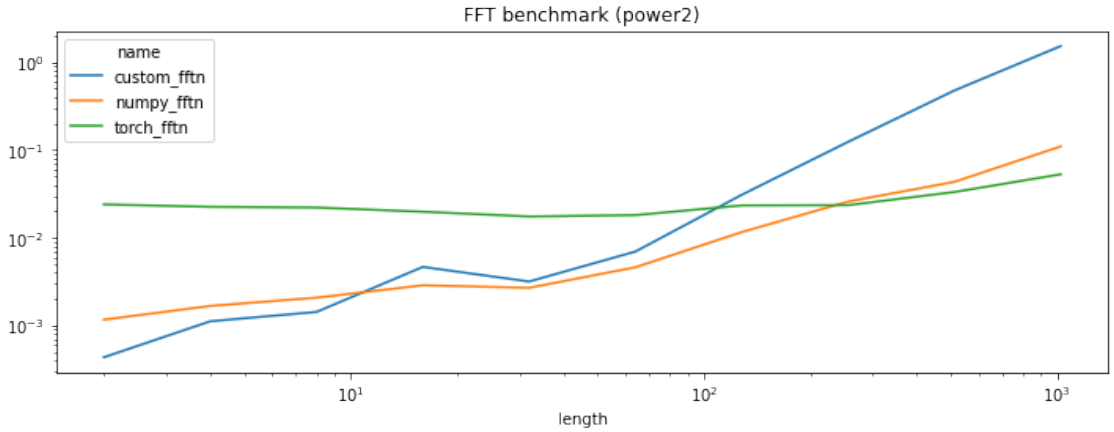
[11]:
```
df = benchmark({'numpy_fftn': numpy_fftn, 'custom_fftn': custom_fftn, 'torch_fftn':␣
↪torch_fftn},
               power2=True)
piv = df.pivot("length", "name", "average")
piv
```

```
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 10/10 [00:13<00:00,  1.33s/it]
```

[11]:

length	custom_fftn	numpy_fftn	torch_fftn
2	0.000434	0.001167	0.023980
4	0.001117	0.001671	0.022530
8	0.001428	0.002077	0.022102
16	0.004654	0.002874	0.019792
32	0.003172	0.002689	0.017474
64	0.006966	0.004612	0.018116
128	0.030904	0.011608	0.023369
256	0.123821	0.025853	0.023532
512	0.476802	0.043352	0.033228
1024	1.527917	0.109868	0.052858

[12]:
```
piv.plot(logy=True, logx=True, title="FFT benchmark (power2)", figsize=(12, 4));
```



1.4 Profiling

[13]:
```
from pyquickhelper.pycode.profiling import profile2graph, profile

shape = [512, 128]
fft_length = [128]
axes = [1]
rnd = numpy.random.randn(*shape) + numpy.random.randn(*shape) * 1j

def f():
    for i in range(100):
        custom_fftn(rnd, 'FFT', fft_length, axes)

stat, text = profile(f)
gr = profile2graph(stat)
print(gr[0].to_text(fct_width=40))
```

```
f                                       --    1    1 -- 0.01752 0.54515 --
<ipython-input-81-3ee1763130c2>:8:f (f)
    custom_fftn                         --  100  100 -- 0.00234 0.52763 --
<ipython-input-7-85a4c9f552d3>:57:custom_fftn (custom_fftn)
        custom_fft                      --  100  100 -- 0.19936 0.52516 --
<ipython-input-7-85a4c9f552d3>:20:custom_fft (custom_fft)
            _dft_cst                    --  100  100 -- 0.31917 0.32366 --
<ipython-input-61-afe90fb073f9>:4:_dft_cst (_dft_cst)
                _arange                 --  200  200 -- 0.00088 0.00449 --
<ipython-input-61-afe90fb073f9>:5:_arange (_arange)
                    <method '…objects> --  200  200 -- 0.00128 0.00128 --
~:0:<method 'astype' of 'numpy.ndarray' objects> (<method 'astype' of
'numpy.ndarray' objects>)
                    <method '…objects> --  200  200 -- 0.00064 0.00064 --
~:0:<method 'reshape' of 'numpy.ndarray' objects> (<method 'reshape' of
'numpy.ndarray' objects>)
                    <built-in…arange> --  200  200 -- 0.00169 0.00169 --
~:0:<built-in method numpy.arange> (<built-in method numpy.arange>) +++
            <built-in met…uiltins.len> --  100  100 -- 0.00011 0.00011 --
~:0:<built-in method builtins.len> (<built-in method builtins.len>) +++
            <method 'toli…ay' objects> --  100  100 -- 0.00024 0.00024 --
~:0:<method 'tolist' of 'numpy.ndarray' objects> (<method 'tolist' of
'numpy.ndarray' objects>)
            <method 'tran…ay' objects> --  100  100 -- 0.00076 0.00076 --
~:0:<method 'transpose' of 'numpy.ndarray' objects> (<method 'transpose' of
'numpy.ndarray' objects>)
            <built-in met…umpy.arange> --  100  100 -- 0.00102 0.00102 --
~:0:<built-in method numpy.arange> (<built-in method numpy.arange>) +++
        <built-in method builtins.len>  --  300  300 -- 0.00013 0.00013 --
~:0:<built-in method builtins.len> (<built-in method builtins.len>) +++
<built-in method builtins.len>          --  400  400 -- 0.00024 0.00024 --
~:0:<built-in method builtins.len> (<built-in method builtins.len>)
<built-in method numpy.arange>          --  300  300 -- 0.00271 0.00271 --
~:0:<built-in method numpy.arange> (<built-in method numpy.arange>)
```

We can see that function `_dft_cst` is the bottle neck and more precisely the exponential. We need to use the symmetries of the matrix it builds.

1.5 Faster __dft__cst

The function builds the matrix $M_{nk} = \left(\exp\left(\frac{-2i\pi nk}{K}\right)\right)_{nk}$ where $1 \leqslant n \leqslant N$ and $1 \leqslant k \leqslant K$. So it computes powers of the unity roots.

$$\exp\left(\frac{-2i\pi nk}{K}\right) = \exp\left(\frac{-2i\pi k}{K}\right)^n = \exp\left(\frac{-2i\pi}{K}\right)^{nk}$$

We use that expression to reduce the number of exponentiels to compute.

[14]:
```
import numpy
from numpy.testing import assert_almost_equal

def _dft_cst(N, fft_length, dtype=numpy.float32):
    def _arange(dim, dtype, resh):
        return numpy.arange(dim).astype(dtype).reshape(resh)

    n = _arange(N, dtype, (-1, 1))
    k = _arange(fft_length, dtype, (1, -1))
    M = (-2j * numpy.pi * k / fft_length) * n
    numpy.exp(M, out=M)
    return M


M = _dft_cst(3, 4, numpy.float32)
M.shape, M.dtype
```

[14]:
```
((3, 4), dtype('complex64'))
```

[15]:
```
M = _dft_cst(4, 3, numpy.float64)
M.shape, M.dtype
```

[15]:
```
((4, 3), dtype('complex128'))
```

[16]:
```
M
```

[16]:
```
array([[ 1. +0.00000000e+00j,  1. +0.00000000e+00j,  1. +0.00000000e+00j],
       [ 1. +0.00000000e+00j, -0.5-8.66025404e-01j, -0.5+8.66025404e-01j],
       [ 1. +0.00000000e+00j, -0.5+8.66025404e-01j, -0.5-8.66025404e-01j],
       [ 1. +0.00000000e+00j,  1. +2.44929360e-16j,  1. +4.89858720e-16j]])
```

[17]:
```
def _dft_cst_power(N, fft_length, dtype=numpy.float32):
    if dtype == numpy.float32:
        ctype = numpy.complex64
    else:
        ctype = numpy.complex128
    M = numpy.empty((N, fft_length), dtype=ctype)
    M[0, :] = 1
    M[1, 0] = 1
    root = numpy.exp(numpy.pi / fft_length * (-2j))
    current = root
    M[1, 1] = root
```

```
    for i in range(2, M.shape[1]):
        current *= root
        M[1, i] = current
    for i in range(2, M.shape[0]):
        numpy.multiply(M[i-1, :], M[1, :], out=M[i, :])
    return M

M_pow = _dft_cst_power(4, 3, numpy.float64)
M_pow
```

[17]:
```
array([[ 1. +0.00000000e+00j,  1. +0.00000000e+00j,  1. +0.00000000e+00j],
       [ 1. +0.00000000e+00j, -0.5-8.66025404e-01j, -0.5+8.66025404e-01j],
       [ 1. +0.00000000e+00j, -0.5+8.66025404e-01j, -0.5-8.66025404e-01j],
       [ 1. +0.00000000e+00j,  1. +6.10622664e-16j,  1. +1.22124533e-15j]])
```

[18]:
```
assert_almost_equal(M, M_pow)
```

[19]:
```
dims = (10, 15)
assert_almost_equal(_dft_cst(*dims, dtype=numpy.float32),
                    _dft_cst_power(*dims, dtype=numpy.float32),
                    decimal=5)
```

1.6 Benchmark again

[20]:
```
def custom_fftn_power(*args, **kwargs):
    return custom_fftn(*args, dft_fct=_dft_cst_power, **kwargs)


%timeit -r 1 -n 1 test_fct(numpy_fftn, custom_fftn_power, decimal=4)
```

```
1.46 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
```

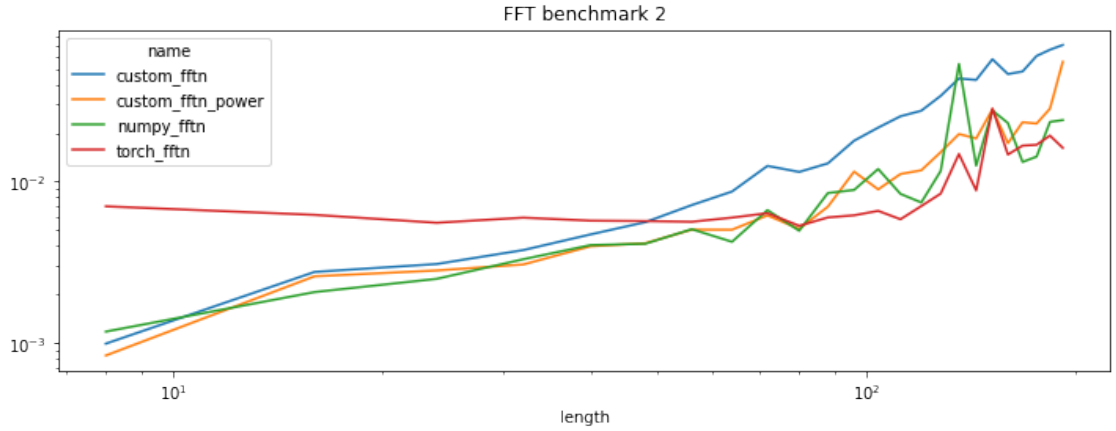
[21]:
```
df = benchmark({
    'numpy_fftn': numpy_fftn, 'torch_fftn': torch_fftn, 'custom_fftn': custom_fftn,
    'custom_fftn_power': custom_fftn_power})
piv = df.pivot("length", "name", "average")
piv[:5]
```

```
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 24/24 [00:07<00:00,  3.19it/s]
```

[21]:

length	custom_fftn	custom_fftn_power	numpy_fftn	torch_fftn
8	0.000991	0.000837	0.001177	0.007033
16	0.002758	0.002591	0.002069	0.006228
24	0.003087	0.002816	0.002499	0.005564
32	0.003767	0.003068	0.003306	0.005985
40	0.004710	0.003975	0.004044	0.005733

[22]:
```
piv.plot(logy=True, logx=True, title="FFT benchmark 2", figsize=(12, 4));
```



[23]:
```
from pyquickhelper.pycode.profiling import profile2graph, profile

shape = [512, 128]
fft_length = [128]
axes = [1]
rnd = numpy.random.randn(*shape) + numpy.random.randn(*shape) * 1j

def f():
    for i in range(100):
        custom_fftn_power(rnd, 'FFT', fft_length, axes)

stat, text = profile(f)
gr = profile2graph(stat)
print(gr[0].to_text(fct_width=40))
```

```
f                                       --    1    1 -- 0.02624 0.57688 --
<ipython-input-92-112d00957d81>:8:f (f)
    custom_fftn_power                   --  100  100 -- 0.00094 0.55064 --
<ipython-input-88-b403af8c0b43>:1:custom_fftn_power (custom_fftn_power)
        custom_fftn                     --  100  100 -- 0.00609 0.54970 --
<ipython-input-7-85a4c9f552d3>:57:custom_fftn (custom_fftn)
            custom_fft                  --  100  100 -- 0.46378 0.54342 --
<ipython-input-7-85a4c9f552d3>:20:custom_fft (custom_fft)
                _dft_cst_power          --  100  100 -- 0.07599 0.07726 --
<ipython-input-85-8502f1ddbe1f>:1:_dft_cst_power (_dft_cst_power)
                    <built-in…y.empty> --  100  100 -- 0.00126 0.00126 --
~:0:<built-in method numpy.empty> (<built-in method numpy.empty>)
                <built-in m…ltins.len> --  100  100 -- 0.00008 0.00008 --
~:0:<built-in method builtins.len> (<built-in method builtins.len>) +++
                <method 'to…' objects> --  100  100 -- 0.00025 0.00025 --
~:0:<method 'tolist' of 'numpy.ndarray' objects> (<method 'tolist' of
'numpy.ndarray' objects>)
                <method 'tr…' objects> --  100  100 -- 0.00096 0.00096 --
~:0:<method 'transpose' of 'numpy.ndarray' objects> (<method 'transpose' of
'numpy.ndarray' objects>)
                <built-in m…py.arange> --  100  100 -- 0.00109 0.00109 --
~:0:<built-in method numpy.arange> (<built-in method numpy.arange>)
```

```
            <built-in met…uiltins.len> --  300  300 -- 0.00020 0.00020 --
~:0:<built-in method builtins.len> (<built-in method builtins.len>) +++
<built-in method builtins.len>           --  400  400 -- 0.00027 0.00027 --
~:0:<built-in method builtins.len> (<built-in method builtins.len>)
```

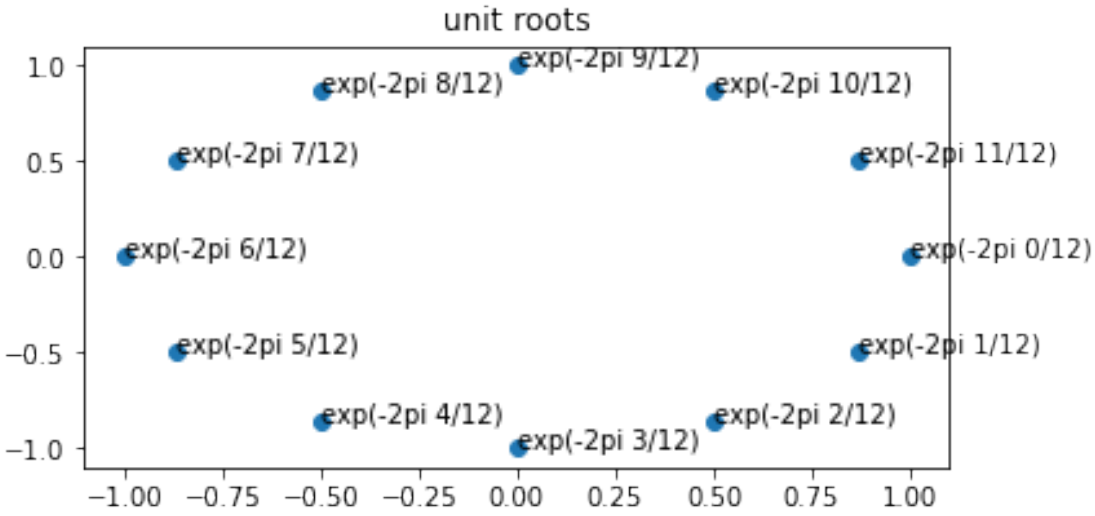
1.7 Cooley–Tukey FFT algorithm

See Cooley–Tukey FFT algorithm.
The FFT matrix is defined by the matrix computation $F_{ak} = X_{an}M_{nk}$, then one coefficient is $(1 \leqslant n, k \leqslant K)$:

$$F_{ak} = \sum_n X_{an}M_{nk} = \sum_n X_{an} \exp\left(\frac{-2i\pi}{K}\right)^{nk}$$

Let's assume K is even, then $\exp\left(\frac{-2i\pi k}{K}\right) = -\exp\left(\frac{-2i\pi(k+\frac{K}{2})}{K}\right)$.

[24]:
```
import matplotlib.pyplot as plt
fig, ax = plt.subplots(1, 1, figsize=(6, 3))
a = numpy.arange(0, 12) * (-2 * numpy.pi / 12)
X = numpy.vstack([numpy.cos(a), numpy.sin(a)]).T
ax.plot(X[:, 0], X[:, 1], 'o');
for i in range(0, 12):
    ax.text(X[i, 0], X[i, 1], "exp(-2pi %d/12)" % i)
ax.set_title('unit roots');
```



Then:

$$\begin{aligned}
F_{a,k+\frac{K}{2}} &= \sum_{n=1}^{N} X_{an} \exp\left(\frac{-2i\pi}{K}\right)^{n(k+\frac{K}{2})} \\
&= \sum_{n=1}^{N} X_{an}(-1)^n \exp\left(\frac{-2i\pi}{K}\right)^{nk} \\
&= \sum_{m=1}^{\frac{N}{2}} X_{a,2m} \exp\left(\frac{-2i\pi}{K}\right)^{2mk} - \sum_{m=1}^{\frac{N}{2}} X_{a,2m-1} \exp\left(\frac{-2i\pi}{K}\right)^{(2m-1)k} \\
&= \sum_{m=1}^{\frac{N}{2}} X_{a,2m} \exp\left(\frac{-2i\pi}{K}\right)^{2mk} - \sum_{m=1}^{\frac{N}{2}} X_{a,2m-1} \exp\left(\frac{-2i\pi}{K}\right)^{2mk} \exp\left(\frac{-2i\pi}{K}\right)^{-k}
\end{aligned}$$

Then:

$$F_{a,k} + F_{a,k+\frac{K}{2}} = 2\sum_{m=1}^{\frac{N}{2}} X_{a,2m} \exp\left(\frac{-2i\pi}{K}\right)^{2mk} = 2\sum_{m=1}^{\frac{N}{2}} X_{a,2m} \exp\left(\frac{-2i\pi}{\frac{K}{2}}\right)^{mk}$$

Finally:

$$\begin{aligned} F_{a,k} &= \sum_{m=1}^{\frac{N}{2}} X_{a,2m} \exp\left(\frac{-2i\pi}{K}\right)^{2mk} + \sum_{m=1}^{\frac{N}{2}} X_{a,2m-1} \exp\left(\frac{-2i\pi}{K}\right)^{2mk} \exp\left(\frac{-2i\pi}{K}\right)^{-k} \\ F_{a,k+\frac{K}{2}} &= \sum_{m=1}^{\frac{N}{2}} X_{a,2m} \exp\left(\frac{-2i\pi}{K}\right)^{2mk} - \sum_{m=1}^{\frac{N}{2}} X_{a,2m-1} \exp\left(\frac{-2i\pi}{K}\right)^{2mk} \exp\left(\frac{-2i\pi}{K}\right)^{-k} \end{aligned}$$

Now, what happen when K is odd, fallback to the original computation.

$$F_{ak} = \sum_n X_{an} M_{nk} = \sum_n X_{an} \exp\left(\frac{-2i\pi}{K}\right)^{nk}$$

[25]:
```
import functools


def cooley_fft_2p(x, fft_length):
    cst = _dft_cst_power(x.shape[-1], fft_length, x.dtype)
    return numpy.matmul(x, cst)


@functools.cache
def _build_fact(p2_2, fft_length, dtype):
    first = numpy.exp(-2j * numpy.pi / fft_length)
    fact = numpy.ones(p2_2, dtype=dtype)
    for k in range(1, p2_2):
        fact[k] = fact[k-1] * first
    return fact.reshape((1, -1))


def build_fact(p2_2, fft_length, dtype):
    return _build_fact(p2_2, fft_length, dtype)


def cooley_fft_recursive(x, fft_length):
    if len(x.shape) != 2:
        raise RuntimeError(
            "Unexpected x.shape=%r." % (x.shape, ))
    dtype = numpy.complex128 if x.dtype == numpy.float64 else numpy.complex64
    if fft_length == 1:
        return x[:, :1].astype(dtype)

    if fft_length % 2 == 0:
        def split(x):
            even = x[:, ::2]
            odd = x[:, 1::2]
            return even, odd

        def tmp1(even, odd, fft_length):
            p2_2 = fft_length // 2
            fft_even = cooley_fft_recursive(even, p2_2)
            fft_odd = cooley_fft_recursive(odd, p2_2)
            return fft_even, fft_odd, p2_2
```

```
        def tmp2(x, fft_even, fft_odd, p2_2):
            fact = build_fact(p2_2, fft_length, fft_even.dtype)

            fact_odd = fft_odd * fact
            return numpy.hstack([fft_even + fact_odd, fft_even - fact_odd])

            # inplace
            # result = numpy.empty((x.shape[0], fft_length), dtype=fft_even.dtype)
            # numpy.multiply(fft_odd, fact, out=result[:, :p2_2])
            # numpy.subtract(fft_even, result[:, :p2_2], out=result[:, p2_2:])
            # numpy.add(fft_even, result[:, :p2_2], out=result[:, :p2_2])
            # return result

        even, odd = split(x)
        fft_even, fft_odd, p2_2 = tmp1(even, odd, fft_length)
        result = tmp2(x, fft_even, fft_odd, p2_2)
    else:
        result = cooley_fft_2p(x, fft_length)

    return result



def cooley_fft(x, fft_length):
    return cooley_fft_recursive(x, fft_length)


def custom_fft_cooley(x, fft_type, length, axis):
    # https://github.com/numpy/numpy/blob/4adc87dff15a247e417d50f10cc4def8e1c17a03/
 ↪numpy/fft/_pocketfft.py#L56
    if fft_type == 'FFT':
        if x.shape[axis] > length:
            # fft_length > shape on the same axis
            # the matrix is shortened
            slices = [slice(None)] * len(x.shape)
            slices[axis] = slice(0, length)
            new_x = x[tuple(slices)]
        elif x.shape[axis] == length:
            new_x = x
        else:
            # other, the matrix is completed with zeros
            shape = list(x.shape)
            shape[axis] = length
            slices = [slice(None)] * len(x.shape)
            slices[axis] = slice(0, length)
            zeros = numpy.zeros(tuple(shape), dtype=x.dtype)
            index = [slice(0, i) for i in x.shape]
            zeros[tuple(index)] = x
            new_x = zeros

        if axis == len(new_x.shape) - 1:
            if len(new_x.shape) != 2:
                xt = new_x.reshape((-1, new_x.shape[-1]))
```

```
            else:
                xt = new_x
            res = cooley_fft(xt, length)
            if len(new_x.shape) != 2:
                res = res.reshape(new_x.shape[:-1] + (-1, ))
        else:
            perm = numpy.arange(len(x.shape)).tolist()
            perm[axis], perm[-1] = perm[-1], perm[axis]
            rest = new_x.transpose(perm)
            shape = rest.shape[:-1]
            rest = rest.reshape((-1, rest.shape[-1]))
            res = cooley_fft(rest, length)
            res = res.reshape(shape + (-1, )).transpose(perm)
            perm[axis], perm[0] = perm[0], perm[axis]
        return res
    raise ValueError("Unexpected value for fft_type=%r." % fft_type)


def custom_fftn_cooley(x, fft_type, fft_length, axes):
    if len(axes) != len(fft_length):
        raise ValueError("Length mismatch axes=%r, fft_length=%r." % (
           axes, fft_length))
    if fft_type == 'FFT':
        res = x
        for i in range(len(fft_length) - 1, -1, -1):
            length = fft_length[i]
            axis = axes[i]
            res = custom_fft_cooley(res, fft_type, length, axis)
        return res
    raise ValueError("Unexpected value for fft_type=%r." % fft_type)


shape = (4, )
fft_length = [3,]
axes = [0]
rnd = numpy.random.randn(*shape) + numpy.random.randn(*shape) * 1j
assert_almost_equal(custom_fftn_cooley(rnd, 'FFT', fft_length, axes),
                    numpy_fftn(rnd, 'FFT', fft_length, axes),
                    decimal=5)
%timeit -n 1 -r 1 test_fct(numpy_fftn, custom_fftn_cooley)
```

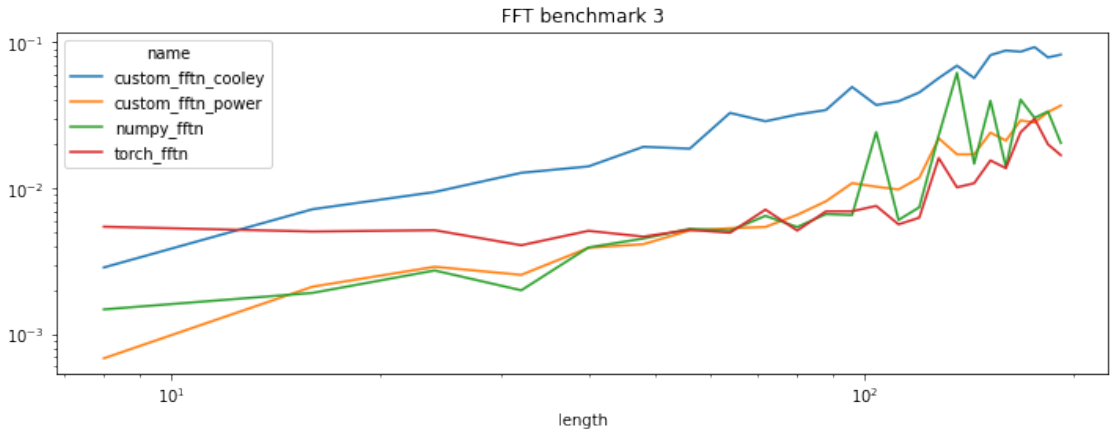
```
1.5 s ± 0 ns per loop (mean ± std. dev. of 1 run, 1 loop each)
```

[26]:
```
df = benchmark({
    'numpy_fftn': numpy_fftn, 'torch_fftn': torch_fftn,
    'custom_fftn_power': custom_fftn_power, 'custom_fftn_cooley': custom_fftn_cooley})
piv = df.pivot("length", "name", "average")
piv[:5]
```

```
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 24/24 [00:10<00:00,  2.35it/s]
```

[26]:
name	custom_fftn_cooley	custom_fftn_power	numpy_fftn	torch_fftn
length				
8	0.002873	0.000685	0.001482	0.005463
16	0.007197	0.002121	0.001922	0.005063
24	0.009443	0.002903	0.002739	0.005169
32	0.012783	0.002556	0.002003	0.004076
40	0.014142	0.003916	0.003937	0.005118

[27]:
```
piv.plot(logy=True, logx=True, title="FFT benchmark 3", figsize=(12, 4));
```

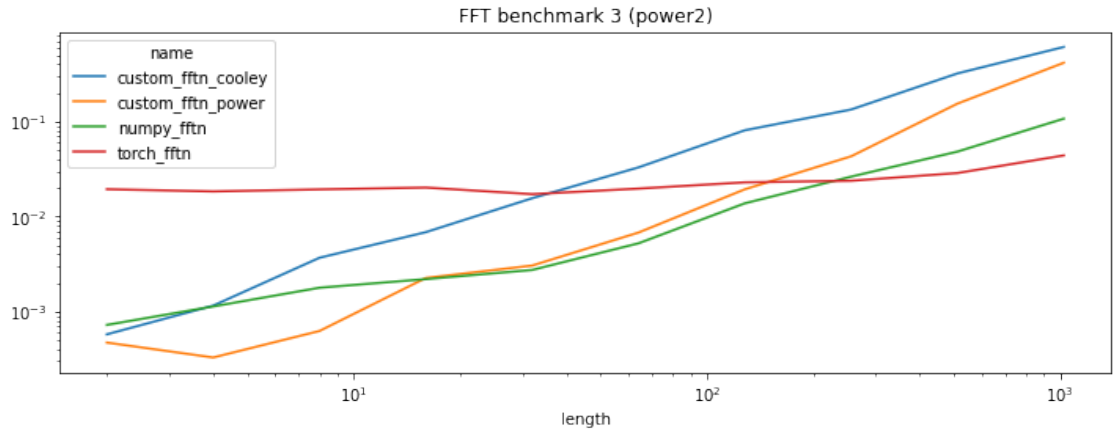


[28]:
```
df = benchmark({
    'numpy_fftn': numpy_fftn, 'torch_fftn': torch_fftn,
    'custom_fftn_power': custom_fftn_power, 'custom_fftn_cooley': custom_fftn_cooley},
    power2=True)
piv = df.pivot("length", "name", "average")
piv[:5]
```

```
100%|¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ ¿ | 10/10 [00:11<00:00,  1.15s/it]
```

[28]:
name	custom_fftn_cooley	custom_fftn_power	numpy_fftn	torch_fftn
length				
2	0.000575	0.000471	0.000722	0.019371
4	0.001153	0.000328	0.001130	0.018366
8	0.003678	0.000624	0.001779	0.019295
16	0.006843	0.002255	0.002192	0.020169
32	0.015574	0.003045	0.002736	0.017193

[29]:
```
piv.plot(logy=True, logx=True, title="FFT benchmark 3 (power2)", figsize=(12, 4));
```



[30]:
```
from pyquickhelper.pycode.profiling import profile2graph, profile

shape = [512, 256]
fft_length = [256]
axes = [1]
rnd = numpy.random.randn(*shape) + numpy.random.randn(*shape) * 1j

def f():
    for i in range(100):
        custom_fftn_cooley(rnd, 'FFT', fft_length, axes)

stat, text = profile(f)
gr = profile2graph(stat)
print(gr[0].to_text(fct_width=40))
```

```
cooley_fft_recursive                    --    100  51100 -- 0.24497 2.68339 --
<ipython-input-139-b9d3f22689f8>:22:cooley_fft_recursive (cooley_fft_recursive)
    split                               --  25500  25500 -- 0.06264 0.06264 --
<ipython-input-139-b9d3f22689f8>:31:split (split)
    tmp1                                --    100  25500 -- 0.09438 2.54540 --
<ipython-input-139-b9d3f22689f8>:36:tmp1 (tmp1)
        cooley_fft_recursive            --  51000    200 -- 0.24336 2.54421 --
<ipython-input-139-b9d3f22689f8>:22:cooley_fft_recursive (cooley_fft_recursive)
+++
    tmp2                                --  25500  25500 -- 0.95948 2.04473 --
<ipython-input-139-b9d3f22689f8>:42:tmp2 (tmp2)
        hstack                          --  25500  25500 -- 0.04799 1.05776 --
<__array_function__ internals>:177:hstack (hstack)
            _vhstack_dispatcher         --  25500  25500 -- 0.02712 0.07002 --
C:/Python395_x64/lib/site-
packages/numpy/core/shape_base.py:218:_vhstack_dispatcher (_vhstack_dispatcher)
                _arrays_for…dispatcher --  25500  25500 -- 0.02361 0.04290 --
C:/Python395_x64/lib/site-
packages/numpy/core/shape_base.py:207:_arrays_for_stack_dispatcher
(_arrays_for_stack_dispatcher)
                    <built-in…hasattr> --  25500  25500 -- 0.01929 0.01929 --
~:0:<built-in method builtins.hasattr> (<built-in method builtins.hasattr>)
```

```
            <built-in met…ay_function> --  25500  25500 -- 0.03753 0.93975 --
~:0:<built-in method numpy.core._multiarray_umath.implement_array_function>
(<built-in method numpy.core._multiarray_umath.implement_array_function>) +++
        build_fact                      --  25500  25500 -- 0.02749 0.02749 --
<ipython-input-139-b9d3f22689f8>:18:build_fact (build_fact)
    <built-in method builtins.len>      --  51100  51100 -- 0.01521 0.01521 --
~:0:<built-in method builtins.len> (<built-in method builtins.len>) +++
    <method 'astype' …darray' objects> --  25600  25600 -- 0.22146 0.22146 --
~:0:<method 'astype' of 'numpy.ndarray' objects> (<method 'astype' of
'numpy.ndarray' objects>)
f                                       --      1      1 -- 0.01449 2.70167 --
<ipython-input-144-55e663ef5e2e>:8:f (f)
    custom_fftn_cooley                  --    100    100 -- 0.00139 2.68718 --
<ipython-input-139-b9d3f22689f8>:112:custom_fftn_cooley (custom_fftn_cooley)
        custom_fft_cooley               --    100    100 -- 0.00135 2.68568 --
<ipython-input-139-b9d3f22689f8>:69:custom_fft_cooley (custom_fft_cooley)
            cooley_fft                  --    100    100 -- 0.00082 2.68421 --
<ipython-input-139-b9d3f22689f8>:65:cooley_fft (cooley_fft)
                cooley_fft_recursive    --    100    100 -- 0.00160 2.68339 --
<ipython-input-139-b9d3f22689f8>:22:cooley_fft_recursive (cooley_fft_recursive)
+++
            <built-in met…uiltins.len> --    300    300 -- 0.00012 0.00012 --
~:0:<built-in method builtins.len> (<built-in method builtins.len>) +++
        <built-in method builtins.len>  --    300    300 -- 0.00011 0.00011 --
~:0:<built-in method builtins.len> (<built-in method builtins.len>) +++
<built-in method builtins.len>          --  77200  77200 -- 0.02367 0.02367 --
~:0:<built-in method builtins.len> (<built-in method builtins.len>)
<built-in method nu…nt_array_function> --  25500  76500 -- 0.58675 0.93975 --
~:0:<built-in method numpy.core._multiarray_umath.implement_array_function>
(<built-in method numpy.core._multiarray_umath.implement_array_function>)
    atleast_1d                          --  25500  25500 -- 0.09562 0.13747 --
C:/Python395_x64/lib/site-packages/numpy/core/shape_base.py:23:atleast_1d
(atleast_1d)
        <method 'append…list' objects> --  51000  51000 -- 0.01708 0.01708 --
~:0:<method 'append' of 'list' objects> (<method 'append' of 'list' objects>)
        <built-in method builtins.len>  --  25500  25500 -- 0.00822 0.00822 --
~:0:<built-in method builtins.len> (<built-in method builtins.len>) +++
        <built-in metho…py.asanyarray> --  51000  51000 -- 0.01655 0.01655 --
~:0:<built-in method numpy.asanyarray> (<built-in method numpy.asanyarray>)
    hstack                              --  25500  25500 -- 0.09871 0.90222 --
C:/Python395_x64/lib/site-packages/numpy/core/shape_base.py:285:hstack (hstack)
        concatenate                     --  25500  25500 -- 0.04882 0.57709 --
<__array_function__ internals>:177:concatenate (concatenate)
            concatenate                 --  25500  25500 -- 0.01049 0.01049 --
C:/Python395_x64/lib/site-packages/numpy/core/multiarray.py:148:concatenate
(concatenate)
            <built-in met…ay_function> --  25500  25500 -- 0.51778 0.51778 --
~:0:<built-in method numpy.core._multiarray_umath.implement_array_function>
(<built-in method numpy.core._multiarray_umath.implement_array_function>) +++
        atleast_1d                      --  25500  25500 -- 0.04022 0.21751 --
<__array_function__ internals>:177:atleast_1d (atleast_1d)
            _atleast_1d_dispatcher      --  25500  25500 -- 0.00838 0.00838 --
C:/Python395_x64/lib/site-
packages/numpy/core/shape_base.py:19:_atleast_1d_dispatcher
```

```
(_atleast_1d_dispatcher)
            <built-in met…ay_function> --  25500  25500 -- 0.03144 0.16891 --
~:0:<built-in method numpy.core._multiarray_umath.implement_array_function>
(<built-in method numpy.core._multiarray_umath.implement_array_function>) +++
        <built-in metho…ns.isinstance> --  25500  25500 -- 0.00892 0.00892 --
~:0:<built-in method builtins.isinstance> (<built-in method
builtins.isinstance>)
```

[31]:

[32]:

[33]: